

# 5.1 Further Integration

## Question Paper

Course	CIEA Level Maths
Section	5. Integration
Topic	5.1 Further Integration
Difficulty	Hard

**Time allowed:** 120  
**Score:** /92  
**Percentage:** /100

**Question 1a**

(a) Use calculus to find

$$\int \left( x^2 + 5 - \frac{3}{x^2} \right) dx$$

[3 marks]

**Question 1b**

(b) Write down

$$\int 3e^{3x} dx$$

[2 marks]

**Question 2a**

(a) Find the integral

$$\int 5(e^{5x} - e^{-5x}) dx$$

[2 marks]

**Question 2b**

(b) Find an expression for  $y$  given that

$$y = \int (\sin x + \cos x) \, dx$$

[2 marks]

**Question 2c**

(c) Use calculus to evaluate

$$\int_{-8}^{-2} \frac{1}{x} \, dx$$

giving your answer as a single logarithm.

[3 marks]

**Question 3a**

(a) Integrate

$$\int 2 \sin x \cos x \, dx$$

[2 marks]

**Question 3b**

(b) Use calculus to find the value of

$$\int_1^3 (4x + 1)^5 dx$$

[4 marks]

**Question 4**

Use a suitable substitution to find the following

$$\int 8x \sin(3x^2 + 1) dx$$

[5 marks]

**Question 5**

Use the substitution  $u = 2 + \ln x$  to show that

$$\int \frac{1}{x(2 + \ln x)^3} dx = \frac{-1}{2(\ln x + 2)^2} + c$$

where  $c$  is the constant of integration.

[5 marks]

**Question 6a**

(a) Show that, for  $\theta \neq k\pi$  (where  $k$  is an integer),

$$\frac{2 - 2 \cos^2 \theta}{\sin 2\theta} = \tan \theta$$

[3 marks]

**Question 6b**

(b) Use calculus and your result from part (a) to show that

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2 - 2 \cos^2 \theta}{\sin 2\theta} d\theta = \frac{1}{2} \ln 3$$

[4 marks]

**Question 7**

Use calculus to find the exact value of

$$\int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} \frac{2 \cos x}{1 - \cos 2x} dx.$$

[6 marks]

**Question 8a**

(a) Find an expression for  $f(x)$  given that

$$f(x) = \int \frac{5}{2x + 3} dx$$

[2 marks]

**Question 8b**

(b) Find an expression for  $y$  given that

$$\frac{dy}{dx} = 2e^{3x-5} - 5e^{5x}$$

[3 marks]

**Question 9a**

(a) Show that

$$\sin^2(3x + 2) \equiv \frac{1}{2}(1 - \cos(6x + 4))$$

[2 marks]

**Question 9b**

(b) Hence, or otherwise, find an expression for

$$\int \sin^2(3x + 2) \, dx$$

[3 marks]

**Question 10a**

(a) Write  $e^{2x}(1 + e^{2x} + e^2)$  in the form  $e^{f(x)} + e^{g(x)} + e^{h(x)}$ , where  $f(x)$ ,  $g(x)$  and  $h(x)$  are all linear functions of  $x$ .



[2 marks]

**Question 10b**

(b) Hence show that

$$\int_0^1 e^{2x}(1 + e^{2x} + e^2) dx = \frac{3}{4}(e^4 - 1)$$

[4 marks]

**Question 11**

Use calculus to show that

$$\int_0^1 \frac{3xe^{-3x^2}}{3 - 2e^{-3x^2}} dx = \frac{1}{4} \ln(3 - 2e^{-3}).$$

[5 marks]

**Question 12a**

(a) Use integration by parts to find

$$\int (2x^2 - 1)e^x dx$$

[5 marks]

**Question 12b**

(b) Show that

$$\int \ln x dx = x \ln x - x + c$$

where  $c$  is the constant of integration.

[4 marks]

**Question 13a**

(a) Express

$$\frac{x^2 - 4x + 7}{(x - 1)(x - 3)^2}$$

as partial fractions.

[4 marks]

**Question 13b**

(b) Hence, or otherwise, find

$$\int \frac{x^2 - 4x + 7}{(x - 1)(x - 3)^2} dx$$

[3 marks]

**Question 14**

Show that

$$\int_0^{\sqrt{\frac{3}{2}}} \frac{4}{9 + 2x^2} dx = \frac{\pi\sqrt{2}}{9}$$

[5 marks]

**Question 15**

Given that  $f'(x) = 2 \tan 3x$  and that  $f(0) = \frac{2}{3}$  show that

$$f(x) = \frac{2}{3}(1 + \ln|\sec 3x|)$$

[4 marks]

**Question 16**

Given

$$f(x) = \int \frac{3(8x^3 + 3x)}{2(4x^4 + 3x^2 + 5)} dx \quad x \in \mathbb{R}$$

and that the graph of  $y = f(x)$  passes through the point  $(1, \ln 144)$  find  $f(x)$ .

[5 marks]

